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Bilevel Design of a Wing Structure Using Response Surfaces

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A methodology for performing bilevel structural optimization of aircraft wing structures is proposed. The overall design problem is decomposed into two levels: a wing (or upper) level and a panel (or lower) level. At the upper level the wing structure is evaluated and designed using a finite element model that is less detailed and contains fewer design variables compared to the actual structure. Constraints on the overall behavior of the structure, such as aeroelastic constraints or constraints on the tip deflection, are imposed at this level. At the lower level more detailed models of certain portions of the structure are used to compute more complex failure modes and to design the details that were not included in the upper-level model. The upper and lower design levels are coordinated with one another using a set of response surface models. Proper coordination between the two levels is maintained through the transfer of stiffness and load information from the upper design level to the lower design level. Results obtained using the proposed technique are presented for a simple wing model.

Introduction

THE optimal design of large aircraft structures presents a challenge to structural designers. The size of the structures, coupled with the large number of structural details that must be modeled, can make the analysis both complex and computationally expensive. One of the ways to make the design task more tractable is to decompose the design problem into two levels: an upper level and a lower level. At the upper design level the overall structure is evaluated and designed using a computational model that is less detailed and contains fewer design variables compared to the actual structure. At the lower level detailed models of certain portions of the structure are used to compute more complex failure modes and to design the details that were not included in the upper level model.

In 1982 Schmit and Mehrinfar[1] proposed a bilevel decomposition technique for the design of a composite wing structure. At the upper level the weight of the structure was minimized subject to strength, deflection, and overall buckling constraints, and at the lower level the change in the stiffness of the skin panels was minimized subject to a set of local buckling constraints. In 1985 Sobieszczanski-Sobieski and coworkers (e.g., Ref. 2) suggested another decomposition procedure based on calculating sensitivity derivatives of the lower-level optima and using them as approximations during the upper-level optimization. Additional references of early work in this area are given by Barthelemy.[3] These and similar decomposition approaches require close integration of the lower-level optimization procedure and the upper-level optimization procedure, which has prevented their ready application with the black-box codes often used in the aircraft industry. Instead, it is common in industry to iterate back and forth between lower and upper design levels without any systematic coordination of the two. This approach typically results in substantial difficulties in converging to an optimal solution.

In the past few years response surface models have emerged as a way to couple optimization procedures efficiently and black-box analysis codes. In this approach the analysis code is executed a large number of times, and a simple approximation is fitted to the results. This approximation is then used as a surrogate model for the analysis in the optimization iterations, obviating the need to integrate the optimizer with the analysis procedure. For example, Vitali et al.[4] and Mason et al.[5] applied the approach in order to connect finite element software without optimization capabilities to an optimizer.

Response surfaces can similarly serve as an easy interface in bilevel optimization schemes. For example, Balabanov et al.[6] used a response surface to perform a combined aerodynamic and structural optimization of a high-speed civil transport configuration. At the upper level aerodynamic analysis was performed, and the aircraft configuration was optimized, whereas at the lower level structural analysis was performed and the overall wing structural weight was optimized. Balabanov et al.[6] performed thousands of wing structural optimizations for different configuration design variables and fitted a quadratic polynomial to the optimal structural weight. This polynomial was then passed on to the configuration optimizer, thus removing the need to couple the structural optimization software to the aerodynamic analysis software directly. Aside from the software integration advantage, the use of response surface approximation for coordinating bilevel optimization has another advantage over integration via derivatives of the lower-level optima. The optimum of a function is often not a smooth function of problem parameters, and this presents problems for optimization procedures. The smoothing associated with the response surface approximation can provide a solution to this problem.

When both upper and lower design levels involve structural models, it is important to formulate the design problem so that the upper-level models and lower-level models remain consistent with one another. For example, if changes in the lower-level design variables

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are allowed to arbitrarily alter the stiffness of the local component then the load paths in the overall structure will change as a result of the lower-level optimization (which will in turn change the loads that are applied to the component). Such changes will create an inconsistency between the upper- and lower-level models because at any step of the iterative design process the optimized local components will exhibit stiffness properties that are different from those that the upper-level optimizer used to determine the component loads. Consequently, it is important to coordinate the flow of information between the two design levels so as to prevent these inconsistencies. The formulations proposed in Refs. 1 and 2 were different attempts to address this problem. Note that this was not an issue in Ref. 6 because there it was assumed that the loads applied to the lower-level structure were a function only of the upper-level aircraft configuration parameters and not on the design of the structure itself.

In the present paper response surface models are used to perform a bilevel structural optimization of a simple wing structure. At the upper level the wing structure as a whole is designed using a simple finite element model. At the lower level the panel weight is minimized, and the details of the stringer cross sections are designed by using a more thorough analysis of the panel response. A new bilevel optimization formulation is explored such that stiffness constraints are imposed on the lower-level optimization problem so as to prevent any inconsistency between the upper- and lower-level design models. A parallel effort (Liu et al.[7]) explores the use of an alternate formulation, where, instead of minimizing the lower-level weight, the lower-level failure load is maximized.

Design Decomposition

The bilevel design procedure is demonstrated using the simple wing model illustrated in Fig. 1. This untapered wing has a 350-in. span and a 30-deg sweep angle. The spar and rib panels were modeled using membrane elements, and the skin panels were modeled using membrane elements and rod elements (representing the stringers). The material properties were assumed to be isotropic with $E = 10.7 \times 10^6$ psi and $\nu = 0.3$. All translational and rotational degrees of freedom at the wing root are restrained, and the wing model is subjected to a single load case, consisting of a 3600-lb upward load applied at the wing tip. Given this load, the design objective is to minimize the total weight of the wing structure subject to strength, panel buckling, and tip deflection constraints. Four of the upper skin panels (marked "Panel 1" near the wing root through "Panel 4" near the tip in Fig. 1) are stiffened panels and will be designed in detail using the bilevel procedure.

If the skin-panel dimensions are small in comparison to the depth of the wing box, the loads on the skin panels will be in the plane of the skin only (N_x, N_y, N_{xy}). These loads will be dictated by the in-plane stiffness of the panel and will not depend on the details of the panel cross section (for example, the exact shape of the stiffeners).

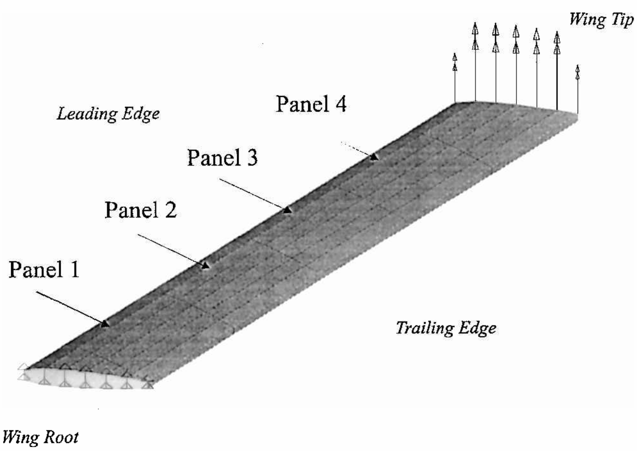

Fig. 1 Upper-level wing finite element model.

Table 1 Upper-level design variables

Identifier	Description
P1, P2, P3, P4	Upper skin thickness
P5, P6, P7, P8, P9, P10	Lower skin thickness
S1, S2, S3, S4, S5, S6	Spar thickness
R1, R2, R3, R4, R5, R6, R7	Rib thickness
Q1, Q2, Q3, Q4	Upper skin stringer area
Q5, Q6, Q7, Q8, Q9, Q10	Lower skin stringer area

This suggests a natural decomposition for the present problem. At the upper level the in-plane stiffness of the panel can be specified, and the in-plane panel loads computed. Given the specified in-plane stiffness properties (or rather requirements) and the corresponding in-plane loads (N_x, N_y, N_{xy}), a detailed lower-level model (such as a detailed buckling load computation model) can be used to design the details of the panel cross section. By iterating back and forth between the upper and lower design levels, a minimum weight wing design can be determined. A more detailed discussion of this iterative procedure is discussed later in the Combined Bilevel Optimization Procedure section.

Upper-Level Design Formulation

Upper-level design variables include the thicknesses of membrane elements (representing the spar, rib, and skin thicknesses) and the areas of rod elements (representing the cross-sectional areas of the stringers). As listed in Table 1 and shown in Fig. 2, there are 33 upper-level design variables: 23 panel thickness design variables and 10 rod area design variables. A lower bound of 0.0125 in. was imposed on all thickness design variables, and a lower bound of 0.050 in.^{2} was imposed on all stringer area design variables. The design optimization problem is as follows:

Given: applied loads
Minimize: total structural weight, w_{tot}
By varying: design variables listed in Table 1
Such that:
1) tip deflection, $\delta \leq 30.0$ in.
2) von Mises stress in spar, rib, and skin elements $\leq \pm 37{,}155$ psi
3) maximum stress in rod elements $\leq \pm 55{,}000$ psi

In addition, it is desired to impose a constraint at the upper level to prevent the top surface panels from buckling. Unfortunately, it is not possible to compute these buckling loads accurately using the upper-level model. Instead, it is necessary to use a computational model that includes the cross-sectional dimensions of each panel and takes these dimensions into account. A design problem is formulated at the lower level to determine these dimensions and meet the buckling requirements.

Lower-Level Design Formulation

Detailed computational models of the top surface skin panels were created using the PASCO (Ref. 8) panel analysis/optimization code. A typical PASCO repeating element is illustrated in Fig. 3. Six design variables were used for the detailed lower-level panel design: the thickness of the skin t_{sk}, the thicknesses of the attachment flange t_{fl}, the thickness t_w and length l_w of the web, and the thickness t_c and length l_c of the stiffener cap. Panels 1–4 have the same overall dimensions: a width of 64.95 in. and a length of 80.82 in. A stringer spacing of 5.413 in. was assumed for all three panels (this translates to 12 stringers per panel). The PASCO models were generated assuming that the lateral edges of the panels (at the skin/spar interface) were simply supported.

Each panel was to be designed such that it does not violate any of the lower-level failure constraints (such as buckling or strength constraints) when subjected to the loads N_x, N_y, and N_{xy}. In addition, each panel is required to have in-plane stiffness properties that match the upper-level stiffnesses dictated by the upper-level dimensions. These specified upper-level stiffnesses are represented by $\bar{A}_{11}$, which describes the "smeared" axial stiffnesses of the panel,

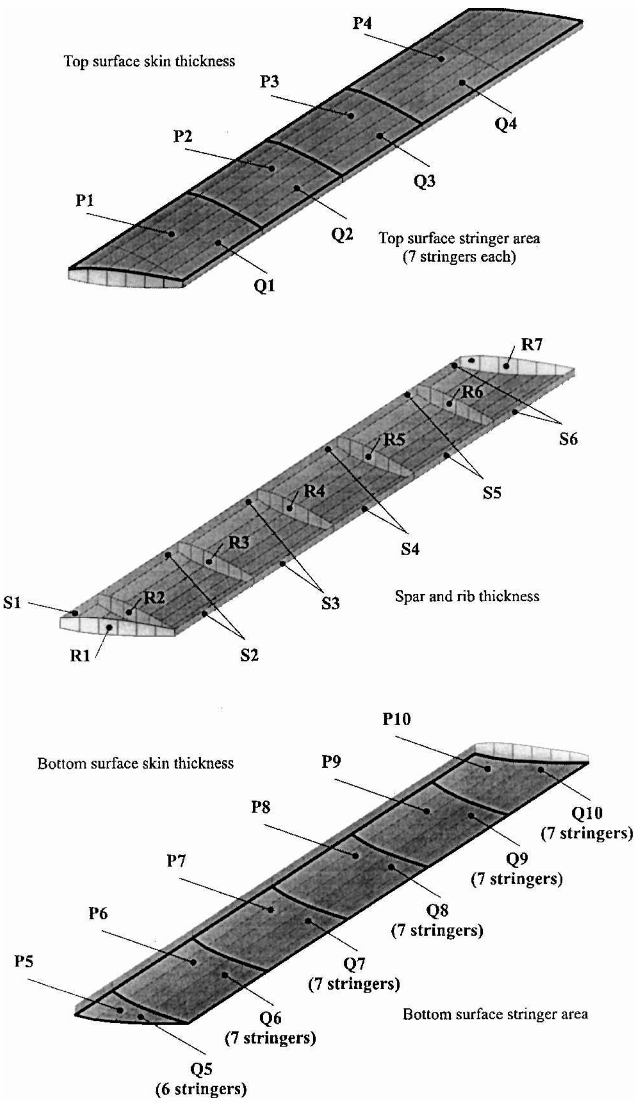

Fig. 2 Upper-level design variables.

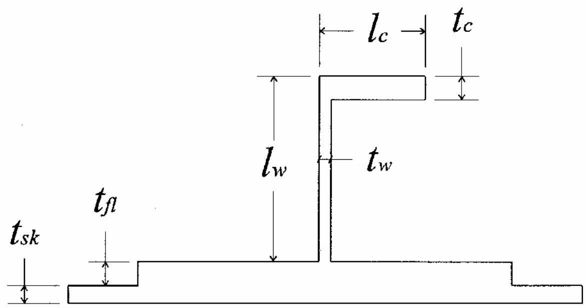

Fig. 3 Typical PASCO repeating element.

and $\bar{A}_{66}$, which describes the smeared shear stiffness of the panel (see Appendix A). Therefore, the lower-level panel optimization is defined as follows:

Given: panel loads N_x, N_y, and N_{xy} and in-plane stiffness parameters $\bar{A}_{11}$ and $\bar{A}_{66}$

Minimize: total lower level panel weight w_l

By varying: t_{sk}, t_{fl}, t_w, l_w, t_c, l_c

Such that:

1) buckling load factor, $\lambda \geq 1.0$

2) smeared in-plane stiffnesses of the panel match the specified $\bar{A}_{11}$ and $\bar{A}_{66}$ values

Bilevel Optimization Procedure

To find the optimum design for the overall structure, it is necessary to iterate back and forth between the upper and lower design levels. For a given upper-level design the upper-level design variables are used to compute the inplane stiffness parameters $\bar{A}_{11}$, $\bar{A}_{66}$, and weight w_g for each panel. Likewise, the upper-level computational model is used to compute the in-plane loads N_x, N_y, and N_{xy} in each of the panels. For each panel that is to be designed at the lower level, the upper-level optimizer then passes these loads and stiffness parameters to the lower design level. The lower-level optimizer then finds the lowest weight design that satisfies the buckling constraints when subjected to the loads N_x, N_y, and N_{xy} and that matches the specified $\bar{A}_{11}$ and $\bar{A}_{66}$ values. The requirement for stiffness matching during the lower-level optimization ensures that the optimized lower-level design will carry the same loads as the corresponding panel in the upper-level design, thus avoiding any load redistribution in the upper-level model as a result of the lower-level optimization.

The upper-level optimizer then compares the optimized weight of the panel obtained using the lower-level optimizer w_l with the weight of the same panel computed using the upper-level design variables w_g and a "weight margin" is computed. If w_l is less than w_g, the weight margin is positive. This means that the panel contains more material than is required in order to satisfy the lower-level constraints. During future iterations, the upper-level optimizer will try to manipulate the upper-level design variables to reduce the weight of this panel. If w_l is greater than w_g, the weight margin is negative. This means that for the given load and stiffness requirements the lower-level design code is unable to generate a design that satisfies the lower-level panel constraints without increasing the amount of material in the panel beyond that present in the upper level model. The upper-level optimizer must manipulate the upper-level design variables in future iterations in order to obtain a positive weight margin for this panel. Therefore, the weight margin can be used to impose a constraint on the upper-level optimization problem that will ensure that the lower-level strength and buckling constraints are satisfied:

$$g_l = 1.0 - w_g/w_l \leq 0.0 \tag{1}$$

where w_g is the weight of the upper-level panel (calculated using the upper-level thickness and area design variables) and w_l is the optimal weight of the lower-level panel computed using the lower-level design variables.

The formulation for the upper-level optimization problem can thus be restated as follows:

Given: applied loads

Minimize: total structural weight w_{tot}

By varying: design variables listed in Table 1

Such that:

1) tip deflection, $\delta \leq 30.0$ in.

2) von Mises stress in spar, rib, and skin elements $\leq \pm 37{,}155$ psi

3) maximum stress in rod elements $\leq \pm 55{,}000$ psi

4) $g_l^n = 1.0 - w_g^n/w_l^n \leq 0.0$; $n = 1, 4$

where w_g^n is the weight of panel n computed using the upper-level design variables and w_l^n is the weight of panel n computed using the lower-level optimization problem.

Bilevel Optimization Interface

In many applications it can prove difficult or time consuming to directly connect the lower-level optimizer to the upper-level optimizer and to coordinate the flow of information between them. An alternative is to approximate the output of the lower-level optimizer (w_l) using one or more simple functions (response surface models) before the upper-level iterations begun. That is, an approximate function can be constructed such that

$$w_l^* = f(N_x, N_y, N_{xy}, \bar{A}_{11}, \bar{A}_{66}) \approx w_l \tag{2}$$

Once the response surface models are constructed, they can be used as surrogates for the lower-level optimizer. For any set of design variables that specify the overall wing design, the wing-design code can use the response surface to obtain an estimate of w_l and use it in Eq. (1) to check whether or not enough material is available at the lower level to satisfy the lower-level failure constraints.

These response surface models are constructed using the lower-level design code before the upper-level design iterations begin. To build the response surface approximation model, first it is necessary to select approximate upper and lower bounds on the in-plane loads N_x, N_y, and N_{xy}, as well as upper and lower bounds on the in-plane stiffness parameters $\bar{A}_{11}$ and $\bar{A}_{66}$ for each of the upper-level panels. These bounds are the maximum and minimum values that each of these parameters is expected to assume during the upper-level optimization process. The lower-level design code is then used to generate a number of optimized (minimum weight) panel designs, each as a function of the loading parameters N_x, N_y, N_{xy}, $\bar{A}_{11}$, and $\bar{A}_{66}$ chosen from within the specified bounds. These designs are then used to generate a response surface approximation to the optimal panel weight as a function of N_x, N_y, N_{xy}, $\bar{A}_{11}$, and $\bar{A}_{66}$. Details of this response surface generation for the present problem are discussed in the next section of this paper.

Because the response surfaces are generated before the bilevel iterations begin, this approach yields certain advantages over the conventional iterative approach. First, a single response surface approximation can be usable for a large number of panels in the structure as long as the general features of the panels are the same. [The panels might still differ from one another by the loads that they carry and/or their stiffnesses.] Next, the designers can ensure that each of the detailed panel designs (and therefore each response surface associated with a particular configuration) is accurate. For example, the designers have the time needed to ensure that each of the detailed panel designs is a true minimum, and not merely a local minimum. Finally, this approach allows for a more compartmentalized approach to the design process, such as might be found in some large organizations. Designers and analysts who specialize in detailed design can direct the construction of the response surfaces and then pass the resulting data to those designers who control the upper-level optimizer. The proposed approach does not suffer from the usual shortcomings of a compartmentalized design approach because during the upper-level design process the optimizer has access to a range of optimal lower-level designs rather than just a single point design.

In addition to the built-in flexibility of the proposed method, it might prove to be more efficient than current design methods. As compared to conventional iterative methodologies, the primary computational expense of the proposed method occurs during the generation of the response surface models and before the upper-level iterations actually begin. Once accurate response surfaces are obtained, the designer can access the optimal panel weight information over and over again at very little additional expense by evaluating a simple response surface function. This will be true as long as changes are not made to the upper-level model that result in significant load or stiffness redistributions which exceed the bounds of the response surface model(s). The computational expense incurred in generating the response surfaces will depend on the efficiency of the chosen lower-level design code and upon the judgment of the designer. If the designer makes inaccurate estimates of the initial load and stiffness bounds for a given response surface, it might be necessary to go back and generate additional lower-level designs before a satisfactory response surface can be obtained.

Response Surface Approximation Model

Response surface methodologies consist of a group of numerical techniques used for constructing empirical models. In general, the methodologies are used to approximate the complex relationship between a set of input variables and an output, or response variable, using a relatively simple functional relationship or model. Because the actual relationship between the input variables and the response variable can be complex (or unknown), the response surface approximation might be valid only for a finite and restricted range of the input variables.

After the response variable has been evaluated at each design point, the resulting data can be fit using an appropriate model, for example, a polynomial function. Once the model is constructed, its accuracy is evaluated and improved as required. The accuracy of a given model can be improved by scaling the input and/or output variables or by eliminating unnecessary input variables. In addition, statistical techniques are available for assessing how well each polynomial coefficient is characterized by the data. Poorly characterized coefficients can be eliminated in order to improve the predictive capabilities of the approximation. If the model's accuracy is still not adequate, it might be necessary to either resort to a different model or to further restrict the range of the input variables.

In the present work a single response surface was constructed for all four upper skin panels. The upper-level finite element model was run for several different wing designs to determine typical values for the panel load and stiffness values. Based on these results, the following load and stiffness ranges were used to construct the response surface approximation (lb/in.):

$$-9000 \le N_x \le 0, \qquad -450 \le N_y \le 0, \qquad 0 \le N_{xy} \le 300$$
$$1.5 \times 10^6 \le \bar{A}_{11} \le 4.0 \times 10^6$$
$$0.15 \times x10^6 \le \bar{A}_{66} \le 1.0 \times 10^6 \qquad (3)$$

Once the ranges have been established, the response variable is evaluated at a number of discrete design points (combinations of the response surface variables) that can be (and typically are) chosen based on the methods of design of experiments. In the present problem the response surface construction is complicated by the fact that not all design points are physically realizable. For example, a design point with a high axial load (N_x) and a low axial stiffness ($\bar{A}_{11}$) might lie outside of the feasible domain of the lower-level design code. One method of working with these infeasible design points is to assign them an artificially high weight (a penalty term). This will prevent the upper-level optimizer from moving into these design regions.

In the present work an initial set of 42 design points were selected using the D-Optimality criterion.[9] The number 42 is arbitrary; it corresponds to twice the number of points required to fit a second-order polynomial in five dimensions. Once the 42 design points were chosen, `PASCO` was run at each point. Each lower-level design was generated subject to constraints on material strength and panel buckling. Because `PASCO` is not capable of imposing equality constraints on the stiffness parameters (only upper and lower bound constraints can be imposed), it was necessary to run `PASCO` up to four times for each design point, each run corresponding to different combinations of the upper and lower bounds on $\bar{A}_{11}$ and $\bar{A}_{66}$ (i-$\bar{A}_{11} \le \bar{A}^*_{11}$, $\bar{A}_{66} \le \bar{A}^*_{66}$; ii-$\bar{A}_{11} \le \bar{A}^*_{11}$, $\bar{A}_{66} \ge \bar{A}^*_{66}$; iii-$\bar{A}_{11} \ge \bar{A}^*_{11}$, $\bar{A}_{66} \le \bar{A}^*_{66}$; and iv-$\bar{A}_{11} \ge \bar{A}^*_{11}$, $\bar{A}_{66} \ge \bar{A}^*_{66}$, where the quantities with an asterisk correspond to the required stiffnesses for a selected design point).

The result of this procedure was the generation of up to four candidate `PASCO` designs corresponding to each attempt to match the stiffnesses of the original design point. In many cases one or more of these candidate designs matched the corresponding original design point stiffness requirement. If, however, `PASCO` was unable to generate a candidate design matching the required stiffnesses, the original design point was marked as infeasible, and its weight was computed using a penalty term (described in Appendix A). Regardless of whether or not the stiffnesses of a given design point was matched by `PASCO`, all unique feasible designs that were generated during this process were retained and used to generate the response surface model. The total number of design points obtained in this manner was approximately 100.

A third-order polynomial was used to obtain an accurate fit to the data. One criterion that was used to assess the quality of fit is a biased measure of the root-mean-square error (e_{rms}) defined as follows:

$$e_{\text{rms}} = \sqrt{\frac{1}{(N-\beta)}\sum_{i=1}^{N}(w-\hat{w})^2} \tag{4}$$

where N is the total number of points considered, β the number of coefficients in the cubic polynomial (in this case 56), w is the exact value of the weight function (determined from PASCO), and $\hat{w}$ is the estimated value of the weight function. The present response surface was created using 104 design points and had an $e_{\text{rms}} = 2.4$ lb. This value can be compared to typical panel weights that range from 80 to 200 lb. Another measure of the quality of fit is the R^2 statistic, which represents the fraction of the variation about the mean that is explained by the fitted model.[9] It has a maximum value of 1.0, with values of R^2 near 1.0 indicating a good fit. R^2 for the present response surface model was calculated to be 0.9993. The R^2 statistic can be adjusted to take into account the number of coefficients in the response surface model; in this form, denoted as R_a^2, the statistic gives an improved indication of the predictive capabilities of the approximation. For the present model $R_a^2 = 0.9986$.

Results

The bilevel methodology was implemented using the Aeroelastic Design Optimization Program ADOP[10] for the upper-level design code. When extracting internal panel loads from the upper-level model during the bilevel iterations, the N_y load on each panel was arbitrarily assumed to be 5% of the corresponding N_x (compressive) load. This was done because the N_y loads that were obtained directly from the upper-level model, although small in comparison to the N_x loads, varied wildly in both magnitude and sign from panel to panel as a result of the coarseness of the upper-level finite element mesh. Additional details of the implementation of the design methodology in the ADOP design environment are presented in Appendix B.

By starting the global-level optimizer at several different points in the design space, two different optimized wing designs, designated design 1 and design 2, were successfully obtained using the proposed method (Table 2). Both designs had similar values for the wing weight, but differed in the distribution of material in the bottom wing surface. Design 1 had relatively thinner skin elements and larger stiffener elements on the bottom surface compared to design 2. (Detailed designs of the panels on the bottom surface were not sought in the present problem formulation.) The final ADOP load, stiffness, and weight values for the top surface panels 1–4 are presented in Table 3 (design 1) and Table 4 (design 2). Note that design 1 and design 2 yield essentially the same results for the four top surface panels. In each case five upper-level constraints were active at the end of the optimization process. These constraints were the tip displacement constraint and the bilevel weight constraint imposed on each of panels 1–4.

To obtain the optimized detailed dimensions for each lower-level panel, a final PASCO optimization was performed for each. These final optimizations were performed at the converged values of the load and stiffness parameters obtained from the upper-level model. For the present problem PASCO was able to generate designs that closely matched the final load and stiffness values for each panel. The optimized upper surface panel designs were essentially the same for both wing designs. The PASCO load, stiffness, and weight values that were achieved for each of panels 1–4 are presented in Table 5, and the corresponding detailed PASCO designs are presented in Table 6. In Table 6 t_{sk} is the thickness of the skin, t_{fl} is the thickness of the attachment (lower) flange, t_w and l_w are the thickness and length of the web, and t_c and l_c are the thickness and length of the stiffener cap. The weight that was predicted by the response surface is w_{rsm}, and w_{pas} is the weight predicted by a single PASCO optimization run (repeated from Table 5). Graphical representations of a repeating element of each of the final panel designs are presented in Fig. 4.

Each of the optimized lower-level panel designs was buckling critical. The critical compressive axial stresses for panels 1–4 were

Table 2 Final values of upper-level design variables (designs 1 and 2)

Identifier	Design 1	Design 2
P1	0.226 in.	0.224 in.
P2	0.183 in.	0.187 in.
P3	0.145 in.	0.145 in.
P4	0.112 in.	0.107 in.
P5	0.250 in.	0.287 in.
P6	0.261 in.	0.322 in.
P7	0.219 in.	0.250 in.
P8	0.167 in.	0.173 in.
P9	0.098 in.	0.099 in.
P10	0.046 in.	0.031 in.
S1	0.0125* in.	0.0125* in.
S2	0.074 in.	0.070 in.
S3	0.050 in.	0.051 in.
S4	0.050 in.	0.051 in.
S5	0.056 in.	0.054 in.
S6	0.050 in.	0.050 in.
Q1	0.961 in.^{2}	0.998 in.^{2}
Q2	0.805 in.^{2}	0.792 in.^{2}
Q3	0.738 in.^{2}	0.744 in.^{2}
Q4	0.641 in.^{2}	0.674 in.^{2}
Q5	0.870 in.^{2}	0.442 in.^{2}
Q6	0.664 in.^{2}	0.094 in.^{2}
Q7	0.307 in.^{2}	0.050* in.^{2}
Q8	0.050* in.^{2}	0.050* in.^{2}
Q9	0.050* in.^{2}	0.050* in.^{2}
Q10	0.050* in.^{2}	0.050* in.^{2}
R1	0.0125* in.	0.0125* in.
R2	0.055 in.	0.061 in.
R3	0.020 in.	0.020 in.
R4	0.0125* in.	0.0125* in.
R5	0.0125* in.	0.0125* in.
R6	0.0125* in.	0.018 in.
R7	0.025 in.	0.027 in.

Table 3 Design 1—final ADOP loads (lb/in.), stiffness parameters (lb/in.), and weights (lb)

Panel	A_{11}	A_{66}	N_x	N_y	N_{xy}	w
1	3.21×10^6	0.93×10^6	−6200	−310	91	158.3
2	2.63×10^6	0.76×10^6	−4754	−238	106	129.5
3	2.16×10^6	0.60×10^6	−3275	−164	71	106.5
4	1.73×10^6	0.46×10^6	−1883	−94	70	85.1

Table 4 Design 2—final ADOP loads (lb/in.), stiffness parameters (lb/in.), and weights (lb)

Panel	A_{11}	A_{66}	N_x	N_y	N_{xy}	w
1	3.22×10^6	0.92×10^6	−6169	−309	89	158.2
2	2.65×10^6	0.77×10^6	−4754	−238	106	130.7
3	2.17×10^6	0.60×10^6	−3264	−163	72	106.5
4	1.70×10^6	0.44×10^6	−1885	−94	66	83.9

Table 5 Final PASCO loads (lb/in.), stiffness parameters (lb/in.), and weights (lb)

Panel	A_{11}	A_{66}	N_x	N_y	N_{xy}	w_{pas}
1	3.21×10^6	0.94×10^6	−6200	−310	91	157.4
2	2.66×10^6	0.76×10^6	−4754	−238	106	130.5
3	2.16×10^6	0.60×10^6	−3275	−164	71	106.0
4	1.73×10^6	0.49×10^6	−1883	−94	70	84.7

Table 6 Final PASCO designs

Panel	t_{sk}, in.	t_{fl}, in.	t_w, in.	l_w, in.	t_c, in.	l_c, in.	w_{rsm}, lb	w_{pas}, lb
1	0.204	0.035	0.072	4.01	0.114	0.803	158.2	157.4
2	0.160	0.034	0.066	3.86	0.120	0.742	129.5	130.5
3	0.122	0.035	0.058	3.68	0.117	0.689	106.5	106.0
4	0.101	0.027	0.041	3.07	0.192	0.488	85.1	84.7

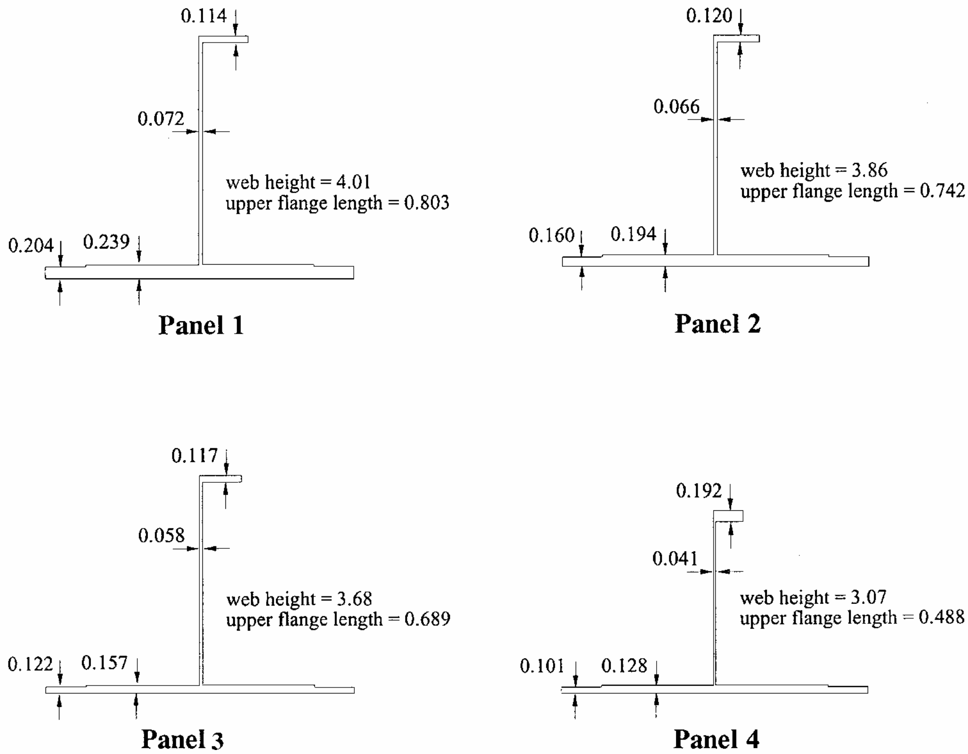

Fig. 4 Optimized panel cross sections (repeating element).

20,800 psi; 19,300 psi; 16,400 psi; and 11,800 psi; respectively. These stresses were considerably below the maximum axial stress allowables for these elements (±55,000 psi in tension/compression). Agreement between the predicted response surface weights and the actual PASCO weights is good in all four cases. All four panels appear to have realistic configurations, although it might be desirable to impose some type of manufacturability constraints on future designs.

Conclusions

A method for performing bilevel design optimization of aircraft wing structures was described. The proposed approach enables designers to account for local structural details, such as stiffener dimensions that require costly analysis computations during the design optimization of the entire structure. Using the finite element code ADOP as a test bed, a simple but flexible interface between the upper and lower design optimization levels was constructed using response surface models. The interface was designed so as to minimize the changes required in either the existing upper-level design codes or the lower-level design codes. Proper coupling was maintained between the upper and lower design levels via the transfer of upper-level stiffness information to the lower-level design code.

The method was demonstrated using a simple isotropic upper-level wing model and the panel design code, PASCO. At the upper level a constraint was imposed on the tip displacement and on element stresses. Using the proposed methodology, four upper skin panels were designed subject to strength and buckling constraints. Lower-level design variables included the thicknesses of skin, web, flange, and cap elements as well as the web height. The resulting panel designs appear to be realistic given the assumptions of the problem.

Appendix A: Calculation of Penalty Terms

When PASCO was unable to converge to a feasible design at or near one of the original design points that will be used for constructing the response surface approximation, the infeasible design point was still retained for use in generating the response surface model. The panel weight used for the infeasible design point was that of an alternate feasible point plus a variable penalty term. The alternate feasible point was chosen from the data that PASCO had generated in the attempt to match the original design point and was chosen to be as near to the original design point as possible. A "nearby" point was defined as a design point that had the same values for the three load parameters as did the original design point and with values for the two stiffness parameters as close as possible to those of the original design point. Typically, the chosen alternate points matched one of the stiffness parameters from the original design point, but not both.

Given the fact that the weight corresponding to the infeasible original design point was not allowed to drop below that of the alternate design point (penalty term was not allowed to be less than zero), an optimization problem was set up to determine the values for the penalty terms. The objective was to minimize the root-mean-square error of the resulting response surface fit to the complete set of data. The design variables were the penalty terms, and except for the requirement that they remain positive there were no additional constraints imposed on the problem. The task was accomplished using the multipurpose optimization code, ADS (Ref. 11). In retrospect, this particular scheme for selecting the design points seems overly complex; other simpler schemes for determining the penalty terms can be devised as well.

Appendix B: Implementation in the ADOP design environment

In the present work the finite element based design code ADOP[10] was used as the upper-level design code. To implement the bilevel methodology in the ADOP design environment, it was necessary to program the proposed weight constraint and response surface interface at the upper level. The inputs to the response surface from the upper-level design code are the panel in-plane loads N_x, N_y, and N_{xy} and the average in-plane stiffnesses $\bar{A}_{11}$ and $\bar{A}_{66}$. Because a panel in the upper-level model is, in general, comprised of a group of several finite elements, it is necessary for the upper-level design code to calculate average load and stiffness values for each panel. A description of the procedures used to calculate the average panel loads, average panel stiffnesses, and constraint derivatives in the case of an isotropic structure can be found in this section.

Average Panel Loads

In a typical ADOP finite element wing model the wing skin is modeled using membrane elements, and the stiffeners are modeled using rod elements. The following procedure was programmed in ADOP to calculate the average panel loads in this case:

First, the average loads in the membrane elements are calculated. These loads represent that portion of the total panel load that is carried in the panel skin. Each ADOP membrane element has four gauss points, and associated with each gauss point is a fixed value of N_x^{ele}, a fixed value of N_y^{ele}, and a fixed value of N_{xy}^{ele}, each in the element coordinate system and in units of pounds/inch. These loads are first converted to the panel coordinate system using user-supplied direction cosines. In the panel coordinate system the x direction is parallel to the stringers, and the y direction is in the plane of the skin. After these element loads have been converted to the panel coordinate system, ADOP searches for the maximum load components in each element. These maximum values are assumed to act over the entire element. The total loads on the panel skin are then obtained by averaging these maximum element loads over the total number of membrane elements in the panel. The resulting N_y^{sk} and N_{xy}^{sk} are equal to the final load values used for the panel N_y^{tot} and N_{xy}^{tot}. The resulting N_x^{sk} value must be combined with the total force in the rod (stringer) elements in order to obtain the total force in the x direction.

The force in the stringers is obtained from the rod elements. Each rod element has an F_x^{ele} value (pounds) associated with it. The total stringer force is then obtained by summing the forces from each rod element in the panel. To combine the F_x^{st} with the N_x^{sk} value, first the membrane N_x^{sk} value is converted to a force by multiplying by the total width of the panel. This skin force can then be added to the stringer force F_x^{st} to obtain a total panel force F_x^{tot}. This force is then converted into N_x^{tot} by dividing by the total panel width. This final value of N_x^{tot}, along with N_y^{tot} and N_{xy}^{tot} just obtained, are passed to the response surface model.

The number of stringers present in the upper-level model for a given panel might differ from the number of stringers present in the lower-level analysis model for that same panel. This will be the case when several stringers are lumped into a single stringer in the upper level model for efficiency reasons. The preceding calculations take into account this possibility.

Average Panel Stiffness

The following procedure was programmed in ADOP to calculate the average panel stiffnesses. For each membrane element $\bar{A}_{11}^{\mathrm{ele}}$ and $\bar{A}_{66}^{\mathrm{ele}}$ are calculated using the following formulas:

$$\bar{A}_{11}^{\mathrm{ele}} = Et_b, \qquad \bar{A}_{66}^{\mathrm{ele}} = Gt_b \tag{B1}$$

where t_b is the thickness of the membrane. These element membrane stiffnesses are averaged over all of the membrane elements in the panel to arrive at the average membrane stiffnesses $\bar{A}_{11}^{\mathrm{sk}}$ and $\bar{A}_{66}^{\mathrm{sk}}$. The skin in each repeating element of the panel will be assumed to have these average stiffness values. Because the stringers are assumed to have no contribution to the shear or transverse stiffnesses, $\bar{A}_{66}^{\mathrm{sk}}$ will be equal to the total panel shear stiffness $\bar{A}_{66}^{\mathrm{tot}}$. The stringer axial stiffness must be added to $\bar{A}_{11}^{\mathrm{sk}}$ in order to obtain the total axial stiffness.

The stringer axial stiffness $\bar{A}_{11}^{\mathrm{st}}$ is computed using the rod elements. For each rod element the axial stiffness is calculated using the following formula:

$$(EA)^{\mathrm{ele}} = ES \tag{B2}$$

where S is the area of the rod element. The average element axial stiffness $(EA)^{\mathrm{avg}}$ is obtained by averaging $(EA)^{\mathrm{ele}}$ over the total number of rod elements in the panel. Each stringer in the upper panel is assumed to have this average axial stiffness. The total stringer axial stiffness for the panel is obtained by multiplying this average stringer stiffness $(EA)^{\mathrm{avg}}$ by the total number of stringers in the panel. This total axial stiffness is then divided by the number of stringers in the lower-level model and the stringer spacing in the lower-level model to obtain the $\bar{A}_{11}^{\mathrm{st}}$ value for a single stringer.

The final $\bar{A}_{11}^{\mathrm{tot}}$ value for the panel is obtained as follows:

$$\bar{A}_{11}^{\mathrm{tot}} = \bar{A}_{11}^{\mathrm{sk}} + \bar{A}_{11}^{\mathrm{st}} = \bar{A}_{11}^{\mathrm{sk}} + (EA)^{\mathrm{avg}}(n_r/n_l b_b) \tag{B3}$$

where b_b is the lower-level panel stringer spacing, n_r is the number of stringers in the upper panel, and n_l is the number of stringers in the lower level panel. $\bar{A}_{11}^{\mathrm{tot}}$ and $\bar{A}_{66}^{\mathrm{tot}}$ are passed to the response surface model.

A variety of different methods for obtaining the average panel forces and stiffnesses from the upper-level finite element model can be envisioned. Alternative methods can be easily incorporated into ADOP if desired.

Constraint Derivatives

To arrive at an optimal solution, the upper-level design code needs to be able to calculate derivatives of the interface constraint with respect to each of the upper-level design variables. The derivative of the interface constraint with respect to the ith design variable $\partial g_l/\partial D_i$ is calculated using finite differences.

The following procedure is followed for each design variable. First, a perturbed value of the design variable D_i' is obtained by incrementing the ith design variable by a small number ΔD_i:

$$D_i' = D_i + \Delta D_i \tag{B4}$$

ADOP then internally calculates the derivatives of the internal loads with respect to the ith design variable $\partial f_{\mathrm{int}}/\partial D_i$. These derivatives are calculated using many of the same subroutines already used to calculate stress derivatives in ADOP. These internal force derivatives are multiplied by ΔD_i in order to obtain the increment in the internal force vector. The updated internal force vector f_{int}' is then obtained as follows:

$$f_{\mathrm{int}}' = f_{\mathrm{int}} + \frac{\partial f_{\mathrm{int}}}{\partial D_i}\Delta D_i \tag{B5}$$

The updated load parameters N_x', N_y', and N_{xy}' can be extracted from this vector. The updated stiffness parameters $\bar{A}_{11}'$ and $\bar{A}_{66}'$, as well as the updated upper-panel weight w_g', are easily obtained in ADOP by calling the original stiffness and weight routines with the perturbed value of the design variable D_i'.

Next, the updated load and stiffness parameters N_x', N_y', N_{xy}', $\bar{A}_{11}'$, and $\bar{A}_{66}'$ are passed to the response surface model, and the updated lower-level panel weight w_l' is returned. The updated interface constraint value g_l' can then be calculated using Eq. (1). Finally, the desired constraint derivative can be calculated as follows:

$$\frac{\partial g_l}{\partial D_i} = \frac{g_l' - g_l}{\Delta D_i} \tag{B6}$$

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References

[1] Schmit, L. A., and Mehrinfar, M., "Multilevel Optimum Design of Structures with Fiber-Composite Stiffened Panel Components," *AIAA Journal*, Vol. 20, No. 1, 1982, pp. 138–147.

[2] Sobieszczanski-Sobieski, J., James, B. B., and Dovi, A. R., "Structural Optimization by Multi-Level Decomposition," *AIAA Journal*, Vol. 23, No. 11, 1985, pp. 1775–1782.

[3] Barthelemy, J.-F. M., "Engineering Design Applications of Multilevel Optimization Methods," *Computer-Aided Optimum Design of Structures: Applications*, edited by C. A. Brebbia and S. Hernandez, Springer-Verlag, New York, 1989, pp. 113–122.

[4] Vitali, R., Park, O., Haftka, R. T., and Sankar, B. V., "Structural Optimization of a Hat Stiffened Panel by Response Surface Techniques," *Proceedings of the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material Conference*, AIAA, Reston, VA, Vol. 4, 1997, pp. 2983–2993.

[5]Mason, B. H., Haftka, R. T., Johnson, E. R., and Farley, G. L., "Variable Complexity Design of Composite Fuselage Frames by Response Surface Techniques," *Thin Wall Structures*, Vol. 32, No. 4, 1998, pp. 235–261.

[6]Balabanov, V. O., Giunta, A. A., Golovidov, O., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Reasonable Design Space Approach to Response Surface Approximation," *Journal of Aircraft*, Vol. 36, No. 1, 1999, pp. 308–315.

[7]Liu, B., Haftka, R. T., and Akgün, M. A., "Two Level Composite Wing Structural Optimization Using Response Surfaces," Struct Multidisc Optim, Vol. 20, Nos. 87–96, 2000.

[8]Stroud, W. J., and Anderson, M. S., "PASCO: Structural Panel Analysis and Sizing Code, Capability and Analytical Foundations," NASA TM-80181, Nov. 1981.

[9]Box, G. P., and Draper, N. R., *Empirical Model-Building and Response Surfaces*, Wiley, New York, 1987, pp. 490–495.

[10]Sharma, A. K., Sikes, G. D., Loikkanen, M. J., and Tzong, T. J., "Aeroelastic Design Optimization Program (ADOP) Users Manual Version 1.1," McDonnell Douglas, Rept. MDC92K0408, St. Louis, MO, Jan. 1993.

[11]Vanderplaats, G. N., "ADS-A FORTRAN Program for Automated Design Synthesis—Version 1.10," NASA CR-177985, Sept. 1985.